THE ENERGY OF MINIMUM DOMINATION IN GRAPHS

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Abstract - The energy calculated in domination graphs is defined in [11]. In this paper, we derived minimum domination cover graphs. For the minimum covering graph found energy, $E_C(G)$ of a graph which depends on its particular minimum cover $C$.

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Keyword - Minimum dominating set, minimum dominating matrix, minimum dominating eigen values, minimum dominating energy of a graph.

I. INTRODUCTION

The mathematical study of Domination Theory in graphs started around 1960. The concept of energy of a graph was introduced by I.Gutman [3,6] in the year 1978. The roots go back to 1862 when C.F. De Jaenisch studied the problem of determining the minimum number of queens necessary to cover an $n \times n$ chess board in such way that every square is attacked by one of the queens. Energy of graphs defined for regular [4], non-regular [9], circulate [12] and random graphs [2], signed graphs in [6] and for weighted graph by I. Gutman and Shao in 2011. In [1] R. Balakrishnan defined the energy of $\pi$ electrons of the molecule is approximately the energy of its molecular graph. The basic properties including various upper and lower bounds for energy of a graph have been established in [7,10], and it has found remarkable chemical application in the molecular orbital theory of conjugated molecules[5,6]. In this paper we derived minimum dominating energy of a graph $E_{D}(G)$ in Cycle graph, Star graph, Wheel graph, Regular graph and also calculate Upper and Lower bounds of energy.

II. PRELIMINARIES

2.1 Labeling

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

2.2 Vertex Labeling

Given a graph $G$, an injective function $f: V(G) \rightarrow N$ has been called a vertex labeling of $G$.

2.3 Edge Labeling

An edge labeling of a graph is a bijection from $E(G)$ to the set $\{1, 2, \ldots, |E(G)|\}$.

2.4 Energy

Energy of a simple graph $G = (V, E)$ with adjacency matrix $A$ is defined as the sum of absolute values of eigen values of $A$ denoted by $E(G) = \sum_{i=1}^{n} |\lambda_i|$ where $\lambda_i$ is an eigen values of $A$, $i = 1, 2, \ldots, n$.

2.5 Dominating set: [11,8]

A subset $S$ of $V(G)$ is said to be dominating set if for every vertex $v$ in $V(G)-S$, there is a vertex $u$ in $S$ such that $u$ is adjacent to $v$. That is a vertex $v$ of $G$ is in $S$ or is adjacent to some vertex of $S$. 

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For instance the vertex set \{b, g\} is a dominating set in this Graph of Fig. 1. The set \{a, b, c, d, f\} is a dominating set of the graph G. For a graph G, G-\{v\} denote the graph obtain by removing vertex v and all edges incident to v.

2.6 Minimum Dominating Set\cite{11}.

A dominating set with least number of vertices is called minimum dominating set. It is denoted as \(\gamma\) set of the graph G.

2.7 Domination Number.\cite{8}

The number of vertices in a minimum dominating set is called domination number of the graph G. It is denoted by \(\gamma(G)\).

2.8 The Minimum Dominating Energy\cite{11}

Let G be a simple graph of order n with vertex set \(V = \{v_1, v_2, \ldots, v_n\}\) and edge set E. A subset D of V is called a dominating set of G if every vertex of V-D is adjacent to some vertex in D. Any dominating set with minimum cardinality is called a minimum dominating set. Let D be a minimum dominating set of a graph G. The minimum matrix of G is the \(n \times n\) matrix defined by

\[
A_{D}(G) = (a_{ij}),
\]

where

\[
a_{ij} = \begin{cases} 
1 & \text{if } v_i v_j \in E, \\
1 & \text{if } i = j \text{ and } v_i \in D, \\
0 & \text{otherwise}. 
\end{cases}
\]

The characteristic polynomial of \(A_{D}(G)\) is denoted by \(f_n (G, \lambda) = \det (\lambda I - A_{D}(G))\). The minimum dominating eigen values of the graph G are the eigen values of \(A_{D}(G)\). Since \(A_{D}(G)\) is real and symmetric, its eigen values are real numbers and we label them in non-increasing order \(\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n\). The minimum dominating energy of G is defined as

\[
E_D(G) = \sum_{i=1}^{n} |\lambda_i|
\]

The trace of \(A_D(G)\) = Dominating Number = k

2.9 Wheel graph

A Wheel graph \(W_n\) is a graph formed by connecting a single vertex to all vertices of a cycle, and otherwise disjoint.

2.10 Star graph

The Star \(S_n\) of order n, sometimes simply know as an n –star, is a tree on n nodes with one node having vertex degree n-1 and the other n-1 havinh vertex degree 1.

2.11 Regular Graph

A regular graph is a graph where each vertex has the same number of neighbors

2.12 Wheel Grah

A Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices can also be defined as the 1-skeleton of an (n-1)-gonal pyramid.
III. MAIN RESULT

Theorem 3.1.
If complete graph $K_n$ satisfies energy of every singleton minimum dominating is constant.

Proof
To prove that the energy of complete graph with minimum dominating sets are constant.
Let $v_1,v_2,v_3,v_4,v_5,...,v_n$ are the vertices of the complete graph $K_n$.
It’s clear that every singleton set in a complete graph is a minimum dominating set, since every vertex of $K_n$ is adjacent to all other vertices of $K_n$.
We compute the adjacency matrix
$$A_D(G) = (a_{ij}),$$
where
$$a_{ij} = \begin{cases} 
1 & \text{if } v_i v_j \in E, \\
1 & \text{if } i = j \text{ and } v_i \in D, \\
0 & \text{otherwise}. 
\end{cases}$$
its eigen values are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. The minimum dominating energy of $G$ is defined as
$$E_D(G) = \sum_{i=1}^{n} |\lambda_i|$$
The trace of $A_D(G) = \text{Dominating Number} = k$
The minimum dominating energy of complete graph is constant

Example 1:
Let $K_5$ be a complete graph with vertex $v_1,v_2,v_3,v_4, v_5$.
The minimum dominating sets are $\{v_1\},\{ v_2\},\{v_3\},\{ v_4\},\{v_5\}$

![Fig 1](image-url)

the minimum dominating adjacency matrix are

$$A_{D_1}(G) = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0
\end{bmatrix}$$
$$A_{D_2}(G) = \begin{bmatrix}
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0
\end{bmatrix}$$
\( A_{D3}(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \)

\( A_{D4}(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \)

The minimum dominating eigen values of the minimum dominating adjacency matrix are \( A_{D1}(G), A_{D2}(G) A_{D3}(G) A_{D4}(G) = -1.0000, -1.0000, -1.0000, -0.2361, 4.2361 \)

The minimum dominating energy are = 7.4722

**Theorem 3.2.**

If circle graph \( C_n \) satisfies energy of minimum dominating set is same value depends upon the minimum dominating adjacent matrix

**Proof**

To prove that a circle graph has same energy in minimum dominating set.

Let \( v_1,v_2,v_3,v_4,v_5,...,v_n \) are the vertices of the circle graph \( C_n \)

Let we take a minimum dominating set

We compute the adjacency matrix

\( A_D(G) = (a_{ij}), \) where

\[
\begin{array}{ll}
1 & \text{if } v_i v_j \in E, \\
1 & \text{if } i = j \text{ and } v_i \in D, \\
0 & \text{otherwise}.
\end{array}
\]

its eigen values are real numbers and we label them in non-increasing order \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \). The minimum dominating energy of G is defined as

\[ E_D(G) = \sum_{i=1}^{n} |\lambda_i| \]

The trace of \( A_D(G) = \) Dominating Number = \( k \)

The minimum dominating energy of complete graph is constant

**Example 2:**

Let \( C_5 \) be a circle graph with vertex \( v_1,v_2,v_3,v_4,v_5 \).

In this graph(fig 2) has the minimum dominating set are \( \{v_1,v_3\} \), \( \{v_1,v_4\} \) is minimal. Then \( (C_5) = 2 \).

The minimum dominating adjacency matrix are
The minimum dominating eigen values of the minimum dominating adjacency matrix are $A_{D1}(G), A_{D2}(G) = -1.4142, -1.1701, 0.688$

The minimum dominating energy of circle graph $C_5 = 7.1686$

**Theorem 3.3.**
Show that the energy of Wheel graphs and Star graphs depends on the centre vertex of G.

**Proof:**
Since the centre vertex of wheel graph and star graph are minimum dominating set of G
Therefore minimum dominating energy of wheel graph and star graph depends on the centre vertex of G.

**Example 3:**
Let $S_9$ be a Star graph with vertex $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$

![Star Graph](image)

In this graph (fig 3) has the minimum dominating set is $\{v_9\}$ is minimal. Then $(S_9) = 1$.

The minimum dominating adjacency matrix are

$$A_{D1}(G) = \begin{bmatrix} 
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix is $A_{D1}(G) = -2.3723, 0, 0, 0, 0, 0, 3.3723$

The minimum dominating energy of Star graph $S_9 = 5.7446$

The energy depends upon the centre vertex of star graph.

**Example 4:**
Let $W_9$ be a Wheel graph with vertex $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$.

In this graph (fig 4) has the minimum dominating set is $\{v_9\}$ is minimal. Then $W_9 = \{1\}$.

The minimum dominating adjacency matrix are

$$A_{D1}(G) =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

The minimum dominating eigen values of the minimum dominating adjacency matrix is

$$A_{D1}(G) = -2.0000, -1.4142, -1.4142, -1.3723, -0.0000, 0.0000, 1.4142, 1.4142, 4.3723$$

The minimum dominating energy of Wheel graph $W_9 = 13.4014$

The energy depends upon the centre vertex of Wheel graph.

IV. Conclusion:

In this paper the minimum domination energy defined of few graphs. The authore extend and to find upper and lower bound of minimum dominating graphs.

Reference: